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RISK-BASED RESOURCE ALLOCATION FOR COLLABORATIVE SYSTEM DESIGN IN A DISTRIBUTED ENVIRONMENT

Risk analysis is important in system design because of its essential role in evaluating functional reliability and mitigating system failures. In this work, we aim at expanding existing risk modeling methods to collaborative system designs: specifically, to facilitate resource allocation among distributed stakeholders. Because of different perspectives and limited local information, inconsistent and/or incoherent risk assessments (such as different probability and confusing consequence evaluations) may occur among stakeholders, who are responsible for same or different risk components of a system. The discrepancies can become potential barriers in achieving consensus or acceptable disagreement for distributed resource allocation. Built upon our previous work, a *Risk-based Distributed Resource Allocation Methodology* (R-DRAM) is developed to help a system manager allocate limited resources among collaborating stakeholders based on a cost-benefit measure of risk. Besides probability and consequence, two additional risk aspects, tolerance and hierarchy, are considered for system risk modeling in a collaborative/distributed environment. Given a total amount of resources to be allocated, the four risk aspects are combined to form the cost-benefit measure in a multi-objective optimization framework for achieving a desired risk reduction of a targeted system. An example is used to demonstrate the implementation process of the methodology. The preliminary investigation shows promise of the R-DRAM as a systematic and quantifiable approach in facilitating distributed resource allocation for collaborative system design.
Keywords: Collaborative System Design, Risk Modeling, Cost Benefit Measure, Distributed Resource Allocation.

1. INTRODUCTION

Collaborative system design involves project planning, concept development, and product implementation by multiple participants. In a global economy, more and more collaborative system designs occur in a distributed environment, where a group of stakeholders at different geographical locations try to develop a product together (shared objectives). Given stringent budgetary constraints and schedule conflicts, resource allocation has become an important task in coordinated effort in real world product development. The focus of this paper is to examine and develop a risk-based resource allocation scheme for collaborative system design with the goal to 1) help system managers to allocate resources effectively and 2) reduce the targeted system's risk level to the maximum degree given limited resource.

Risk, combined with cost, is a crucial factor for making decisions in many real world design problems [1-4]. Theoretically, risk is defined as the combination of probability and consequence that an undesired event may occur [5]. Much effort has been contributed to qualitative or quantitative risk modeling with the assumptions: 1) one single stakeholder oversees the whole system, and/or, 2) the interrelationship among various risks of the system is additive. However, these assumptions may be invalid in a collaborative/distributed environment, where multi-stakeholders are involved, and their inter-connectedness introduces multi-folded complexity to risk analysis. More specifically, the complexity lies in two aspects.

First, inconsistent risk probability and confusing consequence quantification measures may exist. As defined in the previous work [6-7], overlapped risk items refer to those with at least two stakeholders' interests in collaborative system design, while non-overlapped risk items are those concerned by only single stakeholders. Because of different perspectives and limited local information, stakeholders may have different interpretations and evaluations on the same overlapped risk items, resulting in inconsistent probability evaluations. For the consequence measures, conventional risk analysis methods usually quantify consequences by subjective categories and ranks, which are meaningful to the evaluators who defined them, but may not be comprehensible to or agreed by others. Either the probability inconsistency or consequence confusion across distributed stakeholders can lead to inconsistent and incoherent risk assessment for system risk analysis, and therefore, become barriers for collaboration.

Second, risk items can come from two sources: internal and external [5]. In Figure 1, a distributed environment (the largest circle) includes Stakeholder A – X. From the perspective of X (the center circle), a fault tree (within the center circle) can be constructed to indicate the inter-relationship among various risk items. In the fault tree, besides the risk items solely from Stakeholder X, there exist some overlapped risk items from external stakeholders. For example, the risk item E1 exists in X's local fault tree (internal), and it is actually an external risk item from A (arrows are used to indicate the original owners). When such overlapped risk items exist, they can exert consequence on several stakeholders simultaneously. With the confounding consequence effects, overlapped risk items can complicate a system resource allocation strategy if only probability and consequence are used for risk measure. Some existing literatures define risk as a combination of five primitives: outcome, likelihood, significance, causal scenario, and population affected [4]. Inspired by the existing work, two additional risk aspects (besides the probability and consequence), tolerance and hierarchy, are considered for expanding the existing risk modeling to collaborative systematic design.

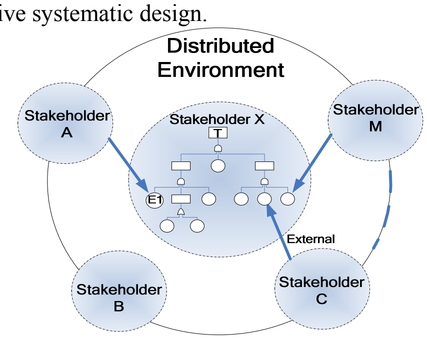

Figure 1: Internal and External Risk Sources

Considering the two aspects, a risk-based design methodology (R-DRAM) is developed to help a system manager allocate a given total amount of resources to distributed stakeholders. A cost benefit measure is introduced to quantify system risk, and a multi-objective optimization framework is adopted to achieve the maximum system risk reduction with uncertainty consideration during the resource allocation.

2. BACKGROUND REVIEW

The research goal is to support collaborative system design based on risk analysis in a distributed environment. Existing work on risk modeling and risk-based system design is reviewed.

2.1 Generic Techniques for Risk Modeling

Various risk definitions are given in different literatures [4-6], which apply for different applications and situations. Accordingly, many qualitative or quantitative risk modeling methods have been developed such as Hazard and Operability Study, Failure Mode and Effects Analysis, Fault Tree Analysis, Event Tree Analysis and so on. [8-10]. Among them, fault tree analysis is widely used in failure analysis and risk-based system design method. A fault tree is a logical graph which shows the relation between system failures. The construction process is: first define an undesirable event, then identify the causal relationships of failures leading to that undesirable event, the numerical probabilities of occurrence can then be entered to evaluate the probability of the events [2-3]. Event tree analysis can illustrate the sequence of outcomes arising after the occurrence of a selected initial event, and it is mainly used in consequence analysis for pre-incident and post-incident application. Fault tree analysis is used in this work with minor adaptations to reflect the relationship between system and failures.

2.2 Function-Based Design Repository

"Design Repository" represents, archives and searches product design knowledge in support of conceptual design activities [11]. It can transform a disparate set of heterogeneous product design knowledge into a single knowledge base, offering extended capability for current Product Data Management applications [2]. While focusing on product functions, design repository considers the relationship between system and component, and uses matrices to store the design knowledge. Figure 2 shows the inter-relationships between system, function failure and components. Several matrices are utilized to represent such relationships. "Product Function Matrix" (PFM) is used to represent the relationship between product and function; "Bill of Material" (BoM) to relate a system and its components; "Function Component Matrix" (EC) to show the connection between functions and components; "Component Failure Matrix" (CF) to relate a component with associated failures; and "Design Structure Matrix" (DSM) to indicate the correlation among the components. By multiplying matrix "EC" and "CF" [12-13], "Function Failure Matrix" (EF) can be obtained to illustrate the relationship between function and failure. Design Repository is meaningful and useful for a wide range of design applications, and also provides a necessary foundation to link the generic risk analysis methods/tools to system design.

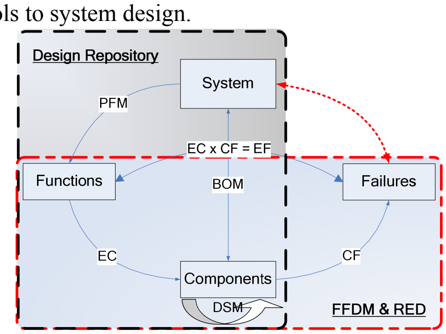

Figure 2: System, Function, Failure, and Component

2.3 Risk Analysis in System Design

Risk-based design is to combine generic risk analysis techniques and design methods for system development and decision making. Risk assessment information and knowledge is obtained and used to guide the design process to yield more reliable products. Stone [12] and Tumer [13] developed a functional basis for functional modeling in product design and yielded a Failure Function Design method (FFDM). FFDM is used for decision making in aerospace system design with the support of a knowledge base in a concurrent design environment [13-14]. As illustrated in Figure 2, FFDM mainly represents the relationship among function, failure, and component using a set of matrices.

Lough [15] developed Risk in Early Design (RED) to map the "EF" to a fever chart for decision making. It is assumed that additive relationship exists among undesirable events associated with components, and then the summation of all component risk is used to indicate the system risk level. Mehr [16] developed a consistent risk-based decision making framework, Risk and Uncertainty Based Integrated Concurrent Design Methodology (RUBIC), for complex aerospace system. RUBIC quantifies risk consequence in monetary unit, and helps final decision making.

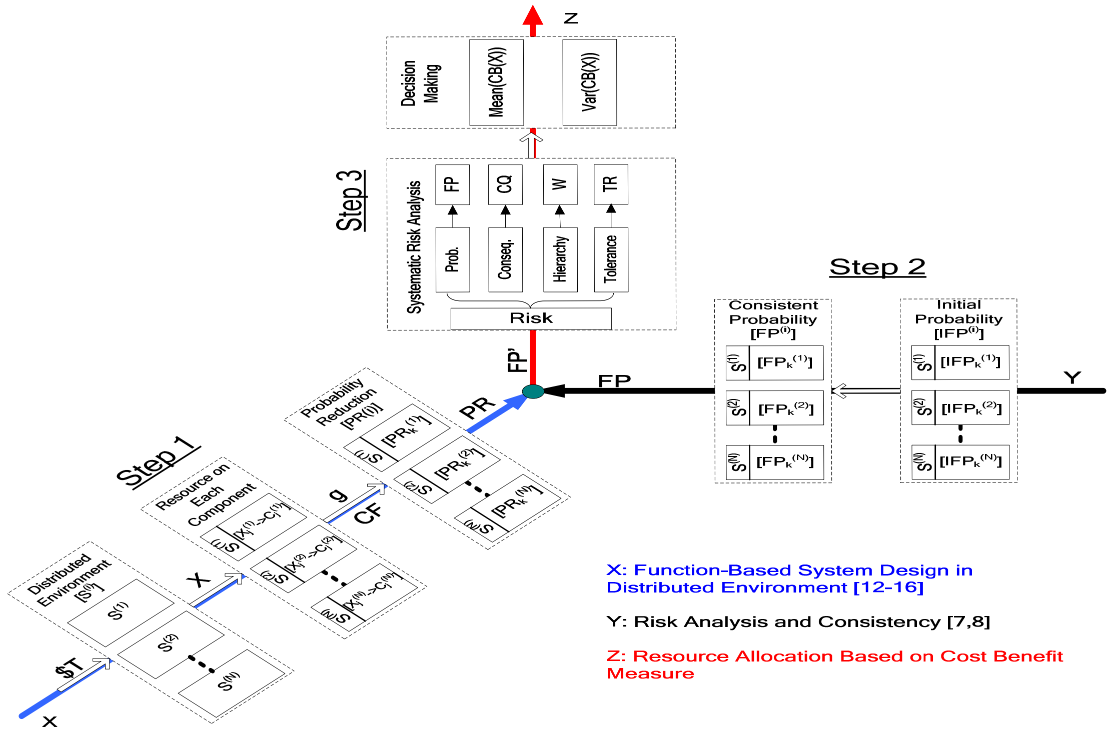

Figure 3: Risk-based Distributed Resource Allocation Methodology (R-DRAM)

Cost-Benefit, in terms of risk reduction and costs of risk analysis, is used as a quantified measure in evaluating the advantage of using Integrated Systems Health Management (ISHM) in aerospace systems [17]. The results show that having ISHM certainly increases the chance of reliable operation and early fault detection. In the existing work, multi-stakeholders scenarios are not considered, and a unanimous decision is assumed.

As a summary, the existing methods have limitations in dealing with risk hierarchical relationship and corresponding influence to system design. They are feasible if independence exists among undesirable events, but such independence condition is sometimes violated in a collaborative/distributed environment due to functional and/or physical inter-dependence among components, and interconnected nature of a distributed network among participating stakeholders. In such cases, the inter-relationship among undesirable events and their impact on the system is important.

3. HIGHLIGHT OF RESEARCH FOCUS

The literature investigation has shown that existing risk analysis methods mostly focus on probability and consequence from the perspective of one stakeholder, and our interest is concerning the potentially different treatment of risk when multiple evaluators involved in a distributed environment. A Risk-based Distributed Resource Allocation methodology (R-DRAM) is introduced to address this concern. A risk-based global coordination scheme focusing on the inconsistent issue of risk probability evaluations has been developed in our previous work [6-7]. Based on this work, we have developed and incorporated the element of resource allocation based on quantitative risk modeling from an overall system perspective. Three important research questions highlight our contributions in the new development:

1) What is the limitation for risk probability reduction in system analysis?

"Without tolerance criteria, you can't make rational risk decisions." [18]. Risk exists anywhere objectively. Its probability can only be mitigated or decreased to certain extent, and additional probability reduction will be unrealistic or require unacceptable cost, thus stakeholders usually assume certain risk tolerance when making risk-based decision. It is good enough to reduce risk probability to the set tolerance rather than pushing for further unrealistic low risk level during resource allocation. In this work, the additional cost benefit beyond the tolerance is set to zero.

2) Is conventional quantification scheme of risk consequence feasible in a distributed environment?

Each stakeholder usually evaluates risk consequence by a qualitative rank. Such a rank is obvious to the stakeholder who defines it, but may not be comprehensible by others in a distributed environment. A uniform consequence measure would be helpful in a distributed environment. Resource allocation is directly associated with money, and risk consequence quantification in monetary unit can provide a globally consistent base for communicating risk information among all stakeholders. Risk has been quantified in monetary unit in business area [19-20], and ISHM related work in engineering design [17]. In this work, this idea is adapted to facilitate the resource allocation in collaborative/distributed system design.

3) How to deal with confounding consequence effects of overlapped risk items?

Overlapped risk items can have confounding consequence effects on several stakeholders. Thus from an overall system perspective, their consequence should include all associated stakeholders' consequences. When allocating resource to mitigate overall system risk, system managers should consider this confounding effect rather than just individual, isolated influence. Besides, some risk items may be dependent on each other, and have different weigh-ins on overall system, so the inter-relationship among risk items is important for system risk analysis.

4. RISK-BASED DISTRIBUTED RESOURCE ALLOCATION METHODOLOGY

A risk-based distributed resource allocation methodology (R-DRAM) is developed to support stakeholders work together effectively and efficiently within acceptable system risk and affordable cost. Figure 3 illustrates the framework of R-DRAM, which includes three parts: 1) Function Based System Design in a Distributed Environment (X axis); 2) Risk Analysis and Consistency (Y axis); 3) Resource Allocation based on a Cost Benefit measure (Z axis). The strategy is to allocate a total amount of available resources on a set of key components instead of abstract functions of a targeted system, and an optimal strategy is pursued to maximize system risk reduction within given constraints [16]. The methodology can be applied to a product design process when the physical form of product is determined. Usually there are a large number of components in the distributed environment, and in the current implementation, only key components associated with important risk items are considered to reduce the computation complexity.

Step 1 (X axis in Figure 3): Function Based System Design in a Distributed Environment

A distributed network structure is used to represent the inter-relationships among stakeholders. Suppose the distributed network includes N (N>1) stakeholders. A stakeholder list **S** can be defined as:

$$\mathbf{S} = [S^{(i)}, i=1...N] \quad (1)$$

where $S^{(i)}$ represents the i^{th} stakeholder in the network.

Each stakeholder first identifies all its key local components, and then a global component list is constructed:

$$\mathbf{C} = [C_j^{(i)}, i=1...N; j = 1...M^{(i)}] \quad (2)$$

where $C_j^{(i)}$ represents the i^{th} stakeholder's j^{th} key components, and $M^{(i)}$ is the number of the i^{th} stakeholder's key components. Each stakeholder has a specific amount of key component, e.g., $M^{(i)}$ components for i^{th} stakeholder. So the total number of key components in the distributed network is: $M=\sum_{l=1}^{n} M^{(l)}$.

A risk space is defined as a set of risk items [6]. In a distributed network, each stakeholder can form a risk space based on its local perspective and available information, and then a global risk space list **F** can be expressed as:

$$\mathbf{F} = [F_j^{(i)}, i=1...N; j=1...K^{(i)}] \quad (3)$$

where $F_j^{(i)}$ represents the i^{th} stakeholder's j^{th} risk item, and $K^{(i)}$ is the number of the i^{th} stakeholder's risk items. The total number of risk items in the distributed network is: $K=\sum_{l=1}^{n} K^{(l)}$.

Then each stakeholder can determine a component-failure matrix $CF^{(i)}$ to indicate the relationship between each component and risk item using Design Repository or related methods [11]. A global component-failure list can then be expressed as:

$$\mathbf{CF} = [CF^{(i)}, i=1...n] \quad (4)$$

where $CF^{(i)}$ represents the i^{th} stakeholder's component-failure matrix including $M^{(i)}$ rows and $K^{(i)}$ columns:

$$CF^{(i)} = [\, CF_{jk}^{(i)}, j = 1...M^{(i)}; k = 1...K^{(i)}] \quad (4')$$

where $CF_{jk}^{(i)}$ indicates the relationship between the j^{th} component and the k^{th} risk item of the i^{th} stakeholder.

$$CF_{jk}^{(i)} = \begin{cases} 1\text{: the } k^{th} \text{ risk item is from the } j^{th} \text{ component} \\ 0\text{: otherwise.} \end{cases}$$

Given the information, a system manager can develop a resource allocation strategy to distribute certain resources to each stakeholder, and further onto its specific key components. Then a resource allocation list **X** can be written as:

$$\mathbf{X} = [x_j^{(i)}, i=1...N; j = 1...M^{(i)}] \quad (5)$$

where $x_j^{(i)}$ is the unknown resource allocated for the i^{th} stakeholder's j^{th} component. To avoid unnecessary superscript and subscript complexity, the resource strategy **X** is mapped to M variables so that for each $x_j^{(i)}$, a corresponding "z_m" (resource allocation variable for component) is constructed:

$$z_m = x_j^{(i)}, \text{ where } m = j + \sum_{l=1}^{i-1} M^{(l)} \quad (5')$$

The expected system risk can be reduced by allocating resources in two ways: reducing risk probability and/or risk consequence. Risk probability reduction is more common, and considered in this paper with the assumption of unchanged risk hierarchy. More resources can decrease risk probability, but the reduction rate depends on specific components and risk items. The true rate is hard or impossible to obtain, but stakeholders can usually estimate a rough relationship between resources and risk probability reduction at a certain confidence interval. For example, a specific component in a stakeholder may have various alternative components with different cost and associated risk probabilities, and then the stakeholder can compare the difference and achieve a mean risk reduction function for each risk item. A probability reduction function list **PR** can be defined as:

$$\mathbf{PR} = [f_j^{(i)}(x), i=1...N; j= 1...K^{(i)}] \quad (6)$$

where $f_j^{(i)}$ is the i^{th} stakeholder's mean risk probability reduction function for the j^{th} risk item. In real world applications, this function is usually non-decreasing: more resources can not yield less risk probability reduction. Also the independent variable of the function $f_j^{(i)}$, i.e., the resource allocated on the j^{th} risk item is $\sum_{k=1}^{M^{(i)}} (CF_{kj}^{(i)} * X_k^{(i)})$, which can also be written in terms of z. Then a new set of function g including independent variable z is constructed for superscript and subscript simplicity:

$$g_m(z) = f_j^{(i)}(z), \text{ where } m = j + \sum_{l=1}^{i-1} K^{(l)} \quad (7)$$

g_m is non-decreasing, and represents a stakeholder' mean risk reduction function. Usually a stakeholder may have a range of evaluations, which can be denoted by an upper bond g_m^+ and lower bound g_m^- as illustrated in Fig. 4.

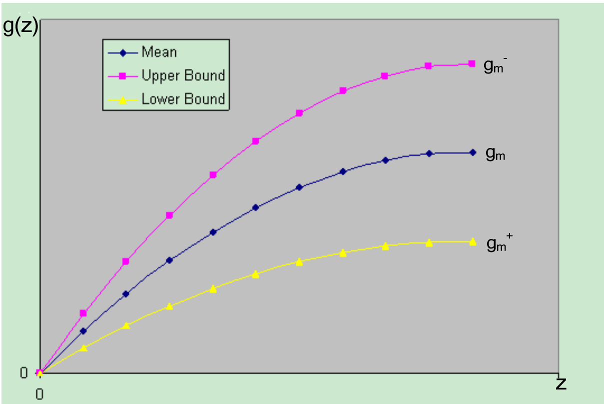

Figure 4: Probability Reduction Function and Its Uncertainty

The lower bound corresponds to the worst case scenario. The real probability reduction function lies between the upper and lower bound statistically, and has a certain distribution. The source of stakeholders' evaluations usually comes from previous historical data, experience, manufacturer's specifications and so on. Existing literature shows that a triangular distribution is a good model for such type of data in the absence of other information [21]. Here, a triangular distribution is assumed and corresponding standard uncertainty function is [21]:

$$u_m(z) = (g_m(z)^+ - g_m(z)^-) / (2\sqrt{6}) \quad (8)$$

Step 2 (Y axis in Figure 3): Risk Analysis and Consistency

Each stakeholder can use existing probabilistic risk analysis methods [8-10] to construct its local risk space **F** (Eq.(3)), and estimate corresponding initial risk probabilities. All stakeholders' probability estimations form a global initial probability list **IFP.** For the overlapped risk items, inconsistency may exist across participating stakeholders. To resolve the inconsistence issue of risk probabilities, a global coordination scheme has been developed in our previous work [6-7]. Globally consistent evaluations among various stakeholders promote mutual understanding, consensus building, and thus lead to better decision making. Suppose a consistent risk probability list **FP** is reached via global coordination, then there exists:

$$\mathbf{FP} = [FP_j^{(i)}, i=1...n; j = 1...K^{(i)}] \quad (9)$$

where $FP_j^{(i)}$ is the i^{th} stakeholder's probability evaluation on the j^{th} risk item, i.e. the probability of $F_j^{(i)}$.

Step 3 (Z axis in Figure 3): Resource Allocation Based on a Cost Benefit Measure

A cost benefit measure of risk is defined as the product of effective global probability reduction and its risk consequence in monetary unit. As illustrated in Figure 3, risk is considered in four aspects to achieve this measure: risk probability, consequence, hierarchy and tolerance.

Steps 1 and 2 focus on the probability aspect, and yield probability reduction function list **PR** in Eq.(6), and an initial consistent risk probability list **FP** in Eq.(9). An updated risk probability list **FP'** can then be expressed in terms of risk reduction function:

$$\mathbf{FP'} = [FP'^{(i)}_j, i=1...n; j = 1...K^{(i)}] \quad (10)$$

where,

$$FP_j'^{(i)}(g_m) = FP_j^{(i)} - PR_j^{(i)} = FP_j^{(i)} - g_m \quad (10')$$

Conventional methods use qualitative ranks to indicate risk consequence. Mehr [16] suggested developing a monetary quantification, but it is hard and time-consuming to obtain such consistent quantified consequence for every risk item in a distributed environment. The proposed strategy here is aimed at the whole system: each stakeholder's overall consequence is quantified rather than every single risk item. Usually a system manager can determine each stakeholder's contribution or weight (based on its importance) to the whole network. Given a total amount of available resources \$T, a quantified consequence set **CQ** can be defined as:

$$\mathbf{CQ} = [CQ^{(i)}, i=1\ldots n] \quad (11)$$

where $CQ^{(i)}$ is the i^{th} stakeholder's consequence, which is the product of \$T and its contribution weight to the whole network.

The expected risk value is the product of risk probability and consequence [2]. In the distributed environment, as stated in Step 3.2, the expected risk for the stakeholder (who "owns" a group of risk items) is of major interest other than single risk item's expected risk value. Each stakeholder's consequence is quantified by Eq.(11), then each stakeholder's risk probability is required, which depends on that stakeholder's risk hierarchy.

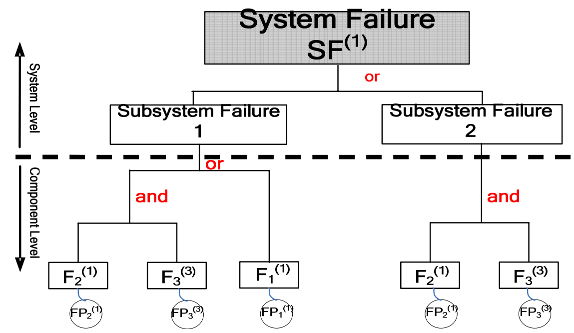

Figure 5: A Single Stakeholder's Fault Tree

A risk hierarchy can illustrate the failure logic among risk items. Existing fault tree techniques can be adopted to represent a risk hierarchy. A fault tree is a simple graphical representation of causal relationships among items using a Boolean expression [2,5]. For the i^{th} stakeholder, a Boolean failure logic $SF^{(i)}$ can be formed. Here, an example is used to illustrate a risk hierarchy in Fig. 5. Stakeholder 1's system failure can be decomposed into two subsystem failures, each of which can be further broken down into more specific risk items with certain causal relationships. Each basic risk item (the smallest rectangles) is originated from a specific component, and accompanies with a specific probability $FP_j^{(i)}$ (shown in a small circle underneath each rectangle).

The risk hierarchy can illustrate the failure logic $SF^{(i)}$ among risk items, and be expressed as:

$$SF^{(1)} = F_1^{(1)} + F_2^{(1)} \bullet F_3^{(3)} + F_2^{(1)} \bullet F_3^{(3)}$$

where "+" represents logic "OR", and "•" indicates "AND". Based on the risk hierarchy, the i^{th} stakeholder's initial and final overall risk probability can be calculated by a risk probability function $w^{(i)}$:

$$w^{(i)}(FP) = Probability(SF^{(i)}) \quad (12)$$

Risk tolerance (TR) refers to a threshold for the system risk reduction where any further reduction would lead to no increase of benefit (i.e., unnecessary reduction). Each stakeholder may set tolerances for all risk items under consideration. From a system perspective, an overall risk tolerance is assigned for each stakeholder. The overall tolerance list **TR** is expressed as:

$$\mathbf{TR} = [TR^{(i)}, i=1...n] \quad (13)$$

where $TR^{(i)}$ represents the i^{th} stakeholder's risk tolerance.

Cost Benefit Measure

In R-DRAM, each stakeholder's cost benefit measure of risk is defined as the product of effective global probability reduction and its risk consequence in monetary unit.

Effective global probability reduction is associated with risk tolerance since addition reduction beyond tolerance is unnecessary. However, risk tolerance imposes many difficulties in calculating the cost benefit, especially for the uncertainty calculation. So here we will first introduce a basic definition and then add in the tolerance consideration with a revised form.

1) A Basic Definition without Tolerance Consideration

An effective probability reduction for the i^{th} stakeholder is:

$$h^{(i)}(g) = w^{(i)}(FP) - w^{(i)}(FP') \quad (14)$$

The initial risk probabilities FP are constant, and FP' are functions in terms of g (See Eq.(10')), thus $h^{(i)'}(g)$ is also a function concerning g. Then the i^{th} stakeholder's mean cost benefit is defined as:

$$CB^{(i)}(g)= CQ^{(i)} * h^{(i)}(g) \quad (15)$$

The summation of all stakeholders' mean cost benefit yields the total mean cost benefit for the distributed network:

$$F(g) = \sum_{i=1}^{n} CB^{(i)}(g) \quad (16)$$

With the existence of a range of estimations for functions g (see Fig. 4), the uncertainty associated with cost benefit is also an important decision factor. Mehr [16] chose variance to model uncertainty, and used correlation matrix to calculate the variance. However, the matrix and necessary expert knowledge may not be available in many distributed environment applications for using this model. In our work, the resource allocation variable z is determined with theoretical zero uncertainty, and g(z) is associated with uncertainty (Eq.(8)); this has rendered the total cost benefit function F(g) (Eq. (16)) with uncertainty consideration. A "Root-Sum-of-Squares" (RSS) method [21] can be utilized to yield the uncertainty associated with the overall cost benefit F(g), and its combined standard uncertainty $u_c(F)$ is then expressed as:

$$u_c^{\,2}(F) = \sum_{i=1}^{N}\left(\frac{\partial F}{\partial g_i}\right)^2 u^2(g_i) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\left(\frac{\partial F}{\partial g_i}\right)\left(\frac{\partial F}{\partial g_j}\right)u(g_i, g_j) \quad (17)$$

where $u(g_i)$ is the standard uncertainty associated with the probability reduction function g_i, which is estimated by Eq.(8); $u(g_i, g_j)$ is the covariance associated with g_i and g_j. Since one resource item z_m may affect several risk items simultaneously, thus g_i and g_j can be correlated. Considering the mathematical probability reduction function in Eq.(7), the correlation factor between any two g_i and g_j is either 1 or 0: for each stakeholder, if two risk items are from the same component, then their probability reduction functions contain the same resource variable resulting in a correlation factor of 1, otherwise the correlation factor is 0. In this way, correlation factor ρ_{ij} between g_i and g_j can be determined, and the covariance associated with g_i and g_j can be calculated as:

$$u(g_i, g_j) = \rho_{ij} * u(g_i) * u(g_j) \quad (18)$$

2) A Revised Definition with Tolerance Consideration

The effective probability reduction may not be always the same as Eq.(14). For instance, if the overall probability is decreased below the set tolerance, then extra probability reduction is unnecessary. It is assumed that the initial $w^{(i)}(FP)$ is always greater than $TR^{(i)}$, otherwise the i^{th} stakeholder's initial risk probability is already below the tolerance, and there is no need for further reduction. Then the system manager can ignore it in resource allocation. With the tolerance consideration, the new effective probability reduction is:

$$h^{(i)}(g)' = \min(w^{(i)}(FP) - w^{(i)}(FP'),\ w^{(i)}(FP) - TR^{(i)}) = \min(h^{(i)}(g), \text{const})) \quad (19)$$

And then the total mean cost benefit for the distributed network can be calculated as:

$$F(g)' = \sum_{i=1}^{n} CQ^{(i)}(g) * h^{(i)}(g)' \quad (20)$$

The associated uncertainty is hard to calculate because of the existence of "minimum" function in Eq. (19). Since only the optimum solutions are of interest and the resource allocation primary objective is to achieve the maximum mean cost benefit, an important conclusion is inferred: given limited resources (total available resource is less than what is needed), no stakeholders' risk can be decreased below the tolerance in the Pareto frontier of the solution domain, i.e., no one can reach its risk tolerance if the strategy is optimum. Because if one stakeholder A or more reaches its tolerance, then the additional resource beyond its tolerance can be taken away and redistributed to other stakeholder B who has not reached its tolerance, and this will lead to the increase of total cost benefit. Thus the original resource allocation strategy is not optimum. So in the Pareto frontier, the mean cost benefit and its uncertainty are the same as Eqs.(16) and (17) because no one reaches its tolerance. For non-optimum solutions, the mean cost benefit is calculated by Eq.(20). Considering the less importance of non-optimum solutions, the induction process for calculating their standard uncertainties is not presented here. The standard uncertainty is estimated by:

$$u_c(F)' = \max\left(u_c(F) + \frac{F(g)' - F(g)}{k}, 0\right) \quad (21)$$

where "k" is the coverage factor chosen on the basis of the desired level of confidence [21]. There is certain distribution associated with the calculated cost benefit, and the worst scenario attracts much attention, and is also an important decision factor. A 5^{th}-percentile cost benefit is chosen to measure the worst scenario, denoted by lower benefit L(g), i.e., the cumulative probability of cost benefit less than L(g) is 5%. L(g) is estimated by:

$$L(g) = F(g)' - k * u_c(F)' \quad (22)$$

Typically k is in the range of 2 to 3. Considering the triangular distribution of all functions g, k is recommended as 2 in this work.

Based on the mean cost benefit and its lower benefit, a resource allocation can be carried out using multi-objective optimization techniques. The objectives are to obtain the maximum mean cost benefit with maximum lower benefit given limited resource \$T. Based on the definition and formulas, the risk-based resource allocation problem is formulated as:

$$\begin{cases} \text{Maximize:} & F(g)' \\ \text{Maximize:} & L(g) \\ \text{s.t.} & \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m^i} x_j^{(i)} = \$T \end{cases} \quad (23)$$

The mathematic problem of the model is a risk-efficient resource allocation problem. The involved functions can be highly complicated, and currently numerical methods are used to achieve optimal solutions.

5. USING AN EXAMPLE TO DEMONSTRATE THE METHODOLOGY

An example is used here to demonstrate the methodology in Section 4. Three stakeholders A, B, and C are involved in a distributed collaborative system design. A system manager administrates this network, and is not satisfied with current expected system risk. Suppose a certain amount of resource is available for mitigating the risk. If the resource is sufficient, then each stakeholder can secure enough resource to decrease their risk to the maximum extent. However, the resource is usually less than what is actually needed, and the system manager must figure out a good strategy to utilize the limited resource.

In the following, the equations are numbered as (*#*), where # is the number of the original equation defined in Section. 4. Stakeholder list **S** can be expressed as:

$\mathbf{S}$ = [A, B, C] (*1*)

Suppose each stakeholder has two key components, then the global component list **C** is:

$\mathbf{C} = [[C_1^{(1)}, C_2^{(1)}], [C_1^{(2)}, C_2^{(2)}], [C_1^{(3)}, C_2^{(3)}]]$ (*2*)

Suppose A and C have three internal risk items, and B has two, then the risk space list is:

$\mathbf{F} = [[F_1^{(1)}, F_2^{(1)}, F_3^{(1)}], [F_1^{(2)}, F_2^{(2)}], [F_1^{(3)}, F_2^{(3)}, F_3^{(3)}]]$ (*3*)

And component-failure matrices are determined:

$\mathbf{CF} = [CF^{(1)}, CF^{(2)}, CF^{(3)}]$ (*4*)

where

$CF^{(1)}$ = [[1,0,1],[0,1,0]]; $CF^{(2)}$ = [[1,0],[0,1]]; $CF^{(3)}$ = [[1,1,0],[0,0,1]]. (*4'*)

Suppose ten-unit resource is available for allocation among A, B, and C. The resource allocation strategy list can be expressed as:

$\mathbf{X} = [[x_1^{(1)}, x_2^{(1)}], [x_1^{(2)}, x_2^{(2)}], [x_1^{(3)}, x_2^{(3)}]]$ (*5*)

A resource allocation variable for component z is defined so that:

$z_1= x_1^{(1)}, z_2= x_2^{(1)}, z_3= x_1^{(2)}, z_4= x_2^{(2)}, z_5= x_1^{(3)}, z_6= x_2^{(3)}$

Three types of risk altitudes are used to simulate stakeholders' evaluations on risk probability reduction: risk prone, risk neutral and risk aversion. Correspondingly their risk reduction functions with boundaries are summarized in Table 1. Suppose all the functions are in the same form: $a*z^2+b*z$.

Suppose the initial consistent risk probability list is:

FP = [[5%, 4%, 3%], [6%, 6%], [5%, 6%, 7%]] (*9*)

Then the new failure probability list is:

FP' = [[5%- g_1, 4%- g_2, 3%- g_3], [6%- g_4, 6%- g_5], [5%- g_6, 6%- g_7, 7%- g_8]] (*10*)

Suppose the quantified consequence for the whole network is $7M, and the contribution weights for three stakeholders are:

CW = [14.3%, 24.6%, 57.1%]

Then the consequence list **CQ** is:

CQ = [$1M, $2M, $4M] (*11*)

Besides the risk probability and consequence, each stakeholder can develop a risk hierarchy for system risk analysis from its local perspective. For instance, stakeholder A can have a risk hierarchy as shown in Fig. 5 (see Section 4). Suppose the following fault Boolean equations can be achieved:

$$\begin{cases} F1 = F_1^{(1)} + F_2^{(1)} \bullet F_3^{(3)} + F_3^{(1)} \bullet F_3^{(3)} \\ F2 = F_1^{(2)} + F_2^{(2)} \bullet F_1^{(1)} \\ F3 = F_1^{(3)} + F_2^{(3)} \bullet F_3^{(3)} \end{cases}$$

Then the global failure probability function can be obtained:

$w^{(1)}(FP) = Pr(F1);\ w^{(2)}(FP)= Pr(F2);\ w^{(3)}(FP)= Pr(F3)$ (*12*)

Suppose all three stakeholders have the same overall risk tolerance of 3%, i.e.

TR = [3%, 3%, 3%] (*13*)

The initial overall probability can be calculated as:

$$\begin{cases} w^{(1)}(FP) = 1-(1-FP_1^{(1)})*(1-FP_2^{(1)}*FP_3^{(3)})*(1-FP_3^{(1)}*FP_3^{(3)}) = 5.46\% \\ w^{(2)}(FP) = 1-(1-FP_1^{(2)})*(1-FP_2^{(2)}*FP_1^{(1)}) = 6.28\% \\ w^{(3)}(FP) = 1-(1-FP_1^{(3)})*(1-FP_2^{(3)}*FP_3^{(3)}) = 5.40\% \end{cases}$$

Given the resource vector, the final overall probability is:

$$\begin{cases} w^{(1)}(FP') = 1-(0.95+ g_1)(1-(0.04- g_2)(0.07- g_8))(1-(0.03- g_3)(0.07- g_8)) \\ w^{(2)}(FP') = 1-(0.94+ g_4)(1-(0.06- g_5)(0.05- g_1)) \\ w^{(3)}(FP') = 1-(0.95+ g_6)(1-(0.06- g_7)(0.07- g_8)) \end{cases}$$

Without tolerance consideration, the effective probability reduction is:

$$\begin{cases} h^{(1)}(g) = w^{(1)}(FP) - w^{(1)}(FP') = -0.9454+(0.95+ g_1)(1-(0.04- g_2)(0.07- g_8))(1-(0.03- g_3)(0.07- g_8)) \\ h^{(2)}(g) = w^{(2)}(FP) - w^{(2)}(FP') = -0.9372+(0.94+ g_4)(1-(0.06- g_5)(0.05- g_1)) \\ h^{(3)}(g) = w^{(3)}(FP) - w^{(3)}(FP') = -0.9460+(0.95+ g_6)(1-(0.06- g_7)(0.07- g_8)) \end{cases} \quad (*14*)$$

So the mean of the total cost benefit is calculated as:

$F(g) = CB(g)= \$1M* h^{(1)}(g) +\$2M* h^{(2)}(g) + \$4M* h^{(3)}(g)$ (*16*)

The standard uncertainty is calculated as:

$$u_c(F) = \sqrt{\sum_{i=1}^{8}\left(\frac{\partial F}{\partial g_i}\right)^2 u^2(g_i) + 2\sum_{i=1}^{8}\sum_{j=i+1}^{8}\left(\frac{\partial F}{\partial g_i}\right)\left(\frac{\partial F}{\partial g_j}\right)\rho_{ij}u(g_i)u(g_j)} \quad (*17*)$$

From the component-failure matrices, for stakeholder A, $F_1^{(1)}$ and $F_3^{(1)}$ are correlated; for stakeholder C, $F_1^{(3)}$ and $F_2^{(3)}$ are correlated. Thus only two corresponding correlation factors are 1 ($\rho_{13}=\rho_{67}=1$), and all the other factors are 0. With the standard uncertainty of each g_i given in Table 1, the cost benefit and its uncertainty can be calculated based on Eqs. (*16*) and (*17*).

With tolerance consideration, the effective probability reduction is:

$$\begin{cases} h^{(1)}(g)' = \min(h^{(1)}(g), 5.46\% - 3\%) \\ h^{(2)}(g)' = \min(h^{(2)}(g), 6.28\% - 3\%) \\ h^{(3)}(g)' = \min(h^{(3)}(g), 5.40\% - 3\%) \end{cases} \quad (*19*)$$

So the mean of the total cost benefit is calculated as:

$F'(g) = \$1M* h^{(1)'}(g) +\$2M* h^{(2)'}(g) + \$4M* h^{(3)'}(g)$ (*20*)

he standard uncertainty is calculated as:

$$u_c(F)' = \max(u_c(F) + \frac{F(g)' - F(g)}{k}, 0) \quad (*21*)$$

The lower benefit can be calculated as:

$L(g) = F(g)' - 2* u_c(F)'$ (*22*)

Table 1: Risk Reduction Functions

	$f_1^{(1)}(x_1^{(1)})$ $[g_1(z_1)]$	$f_2^{(1)}(x_2^{(1)})$ $[g_2(z_2)]$	$f_3^{(1)}(x_1^{(1)})$ $[g_3(z_1)]$	$f_1^{(2)}(x_1^{(2)})$ $[g_4(z_3)]$	$f_2^{(2)}(x_2^{(2)})$ $[g_5(z_4)]$	$f_1^{(3)}(x_1^{(3)})$ $[g_6(z_5)]$	$f_2^{(3)}(x_1^{(3)})$ $[g_7(z_5)]$	$f_3^{(3)}(x_2^{(3)})$ $[g_8(z_6)]$
Altitude	Prone	Neutral	Aversion	Prone	Neutral	Aversion	Prone	Neutral
a	-0.00036	0	0.00016	-0.00048	0	0.00016	-0.00024	0
b	0.0072	0.002	0	0.0096	0.003	0	0.0048	0.002
g^+	1.3 g_1	1.2 g_2	1.1 g_3	1.3 g_4	1.2 g_5	1.1 g_6	1.3 g_7	1.2 g_8
g^-	0.7 g_1	0.8 g_2	0.9 g_3	0.7 g_4	0.8 g_5	0.9 g_6	0.7 g_7	0.8 g_8
u_m	0.1225 g_1	0.0815 g_2	0.041 g_3	0.1225 g_4	0.0815 g_5	0.041 g_6	0.1225 g_7	0.0815 g_8

Finally, a multi-objective optimization design model can be constructed as the following:

$$\begin{cases} \text{Maximize:} \quad F(g)' \\ \text{Maximize:} \quad L(g) \qquad\qquad (*23*) \\ \text{s.t.} \quad \sum_{i=1}^{3}\sum_{j=1}^{2} X_j^i = 10 \end{cases}$$

Both F(g)' and L(g) are complicated functions in terms of g, and numerical methods are used to solve this multi-objective model. An enumeration of all strategies X is possible with the assumption of integer $X_j^{(i)}$. The solution process is implemented in Excel. Each strategy corresponds to a specific mean, lower benefit and associated uncertainty of cost benefit measure. Figure 6 shows the trade-off between mean and standard uncertainty, also a Pareto frontier is determined. Based on the frontier, the lower benefit is calculated and drawn in Fig. 6.

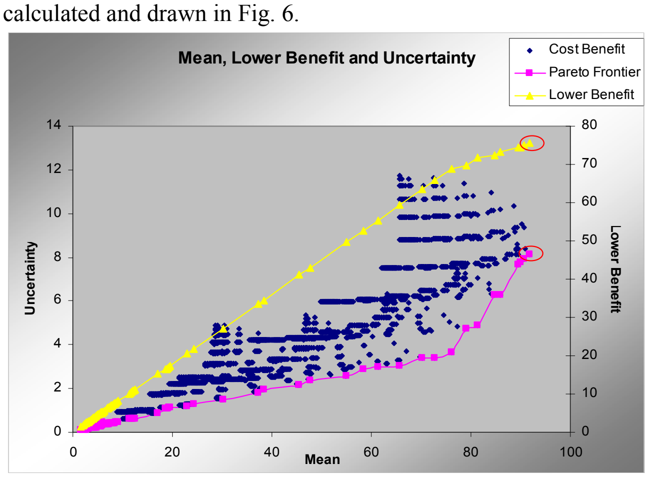

Figure 6: Cost Benefit Mean, Lower Benefit and Uncertainty

For this simple case, the point marked in the circle represents the maximum mean with biggest lower benefit value, i.e. the most optimum strategy with the mean and lower benefit of \$91,900 and \$75,600 respectively. The resource allocation list X is:

$x^{(1)} = [\,4\,,0\,]$; $x^{(2)} = [\,4\,,0\,]$; $x^{(3)} = [\,2\,,0\,]$. (*24*)

The maximum mean cost benefit does not always accompany with the biggest lower benefit. For different cases, there may be trade-off between mean and lower benefit of the cost benefit. The system manager may need to choose the appropriate point in the Pareto frontier.

To verify the standard uncertainty and lower benefit calculation, a Monte-Carlo simulation is used, and both results agree well. For example, for the optimum strategy chosen in Eq.(*24*), 1000 simulations are used to simulate the triangular distribution of each probability reduction function g_i, and then for each simulation, the corresponding cost benefit is calculated based on Eq.(16) without tolerance consideration and Eq. (20) with tolerance consideration. Thus the mean cost benefit for both scenarios can be calculated, and a distribution for the cost benefit measure can be obtained as shown in Fig.7. Note that, "PDF" represents probability density function.

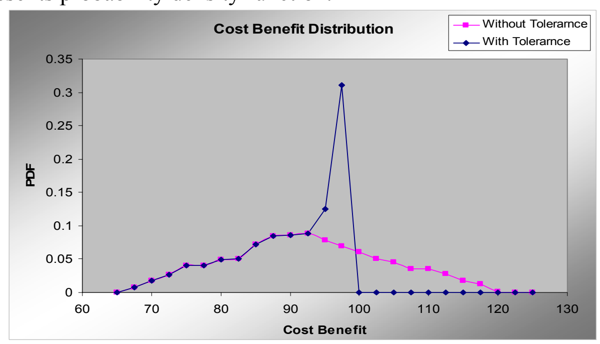

Figure 7: Distribution of Cost Benefit Based on 1000 Monte-Carlo Simulations

As shown in Figure. 7, without tolerance consideration, the cost benefit applies to a rough triangular distribution. The mean, lower and upper bound are approximately \$92,000, \$66,000 and \$118,000 respectively, also the 5th-percentile lower benefit is \$75,000. When considering tolerance, the minimum function in Eq.(20) leads to a maximum mean value, and thus the probability density over this value is zero. Its mean, lower and upper bound are approximately \$89,000, \$66,000 and \$96,000 respectively, also the 5th-percentile lower benefit is \$75,000. The important finding is that in the lower range of cost benefit, the probability densities for both cases are the same, and their lower benefits are also the same, which leads to the formation of Eq.(21). It can be verified that the calculated mean cost benefit and its lower benefit from these Equations are within 5% variation of those from Monte-Carlo simulation. The small variation of mean cost benefit between equation calculation and Monte-Carlo simulation comes from some correlation between functions g, and a more complicated and accurate method may be needed to calculate the expected value of Eq.(20). But considering the calculation complexity and the small variation, no correlation factors are considered when calculating mean value in this example.

6. CONCLUSIONS

Facing fierce competition and shrinking resources, today's global economy requires strategic resource allocation to ensure success of a collaborative design of systems. Risk has become an important criterion for system managers to allocate limited resources to multiple stakeholders. In this work, a risk-based distributed resource allocation methodology (R-DRAM) is developed with a stakeholder- and component-oriented resource allocation strategy. A theoretical framework of a resource allocation mechanism based on a cost benefit measure of risk is established with three major steps: 1) function-based system design in a distributed environment from existing work [11-15]; 2) consistent risk analysis through global coordination based on previous work [6-7]; 3) distributed resource allocation based on cost benefit measure from a system failure view. An example is used to demonstrate the working process and implementation details. A Monte-Carlo simulation corroborates the findings.

The advantages include: 1) two additional risk aspects besides the conventional probability and consequence are considered to achieve more comprehensive quantitative risk modeling where multiple-participants are involved; 2) risk is quantified in monetary unit with uncertainty modeling, which provides a more streamlined treatment than solely experience-based risk uncertainty analysis; 3) R-DRAM can be used not only to distribute resource, but also to determine effective resource needed for a distribute network; 4) R-DRAM can deal with degraded or redundant functionalities in product hierarchy. Stakeholders' risk hierarchies can reflect such functionalities, and the proposed strategy can allocate appropriate resources on the components related to these functionalities.

There are several areas currently under consideration for future development. More streamlined methods with better computational efficiency are an interest of our future work. The functions used in the current model (Eqs. 16-21) entail a certain level of computation complexity, and numerical methods are used to achieve the solutions with the assumptions of all integer resource numbers. The current framework of R-DRAM focuses on rational decision making based on quantitative risk analysis in product design, and human/social factors are not considered, which may be very important in real world applications. In addition, different types of examples and real world applications are needed to test and improve the current work.

REFERENCES

[1] Tumer, I.Y., Barrientos, F.A., and Mehr, A.F., 2005, "Towards Risk Based Design (RBD) of Space Exploration Missions: a Review of RBD Practice and Research Trends at NASA," Proceedings of the 2005 ASME Design Engineering Technical Conferences, DETC2005-85100, Long Beach, CA.

[2] Haimes, Y.Y., 1998, Risk Modeling, Assessment, and Management, John Wiley & Sons, Inc., New York.

[3] Melchers, R.E. and Stewart, M.G., 1995, Integrated Risk Assessment, A.A. Balkema, Brookfield.

[4] Kumamoto, H., Henley, E.J., 1996, Probabilistic Risk Assessment and Management for Engineering and Science, IEEE Press, New York.

[5] Bedford, T., and Cooke, R., 2001, Probabilistic Risk Analysis: Foundations and Methods, Cambridge University Press, Boston.

[6] Qiu, Y. M., Ge, P., and Yim, S. C., 2006, "A Risk-Based Approach to Support Global Coordination in Distributed Environment," Proceeding of the 2006 ASME Design Engineering Technical Conferences, DETC2006-99158, Pittsburgh, PA; in print, Concurrent Engineering: Research and Applications (CERA), 2007.

[7] Qiu, Y. M., Ge, P., and Yim, S. C., 2006, "Enabling Local Risk Assessment to Support Global Collaboration in Distributed Environment," Proceeding of the 2006 ASME Design Engineering Technical Conferences, DETC2006-99159, Pittsburgh, PA; in print, Concurrent Engineering: Research and Applications (CERA), 2007.

[8] Sutton, I.S., 1992, Process Reliability and Risk Management, 1st Ed. Van Nostrand Reinhold.

[9] Suokas, J., and Rouhiainen, V., 1993, Quality Management of Safety and Risk Analysis, Elsevier Science Publishers B.V.

[10] Andrews, J. D., and Moss, T.R., 1993, Reliability and Risk Assessment, 1st Ed. Longman Group, UK.

[11] Bohm, M., Stone, R. and Szykman, S., 2005, "Enhancing Virtual Product Representations for Advanced Design Repository Systems," Journal of Computer and Information Science in Engineering, 5(4), pp.360-372.

[12] Stone, R., and Tumer, I., 2005, "Function Based Risk Assessment: Mapping Function to Likelihood," Proceeding of the 2005 ASME Design Engineering Technical Conferences, DETC2005-85053, Long Beach, CA.

[13] Tumer, I. and Stone, R., 2003, "Mapping Function to Failure during High-risk Component Development," Journal of Research in Engineering Design, 14, pp.25-33.

[14] Mehr, A.F., and Tumer, I.Y., 2005, "A New Approach to Probabilistic Risk Analysis in Concurrent and Distributed Design of Aerospace Systems," Proceeding of the 2005 ASME Design Engineering Technical Conferences, DETC2005-35056, Long Beach, CA.

[15] Lough, K. G., Stone, R. B., and Tumer, I., 2006, "The Risk in Early Design (RED) Method: Likelihood and Consequence Formulations," Proceeding of the 2006 ASME Design Engineering Technical Conferences, DETC2006-99375, Pittsburgh, PA.

[16] Mehr, A. F., and Tumer, I., 2006, "Risk-based Decision Making for Managing Resources during the Design of Complex Space Exploration Systems," Journal of Mechanical Design, 128, pp.1014–1022.

[17] Hoyle, C., Mehr, A.F., Tumer, I.Y., and Chen, W., 2007, "Cost Benefit Quantification of ISHM in Aerospace Systems," Proceeding of the 2007 ASME Design Engineering Technical Conferences, DETC2007-35412, Las Vegas, Nevada.

[18] Bendixen, L.M., 1987, "Risk Acceptability Criteria," ASCE Spring Convention &amp, Energy Conference.

[19] Rockafellar, R.T., and Uryasev, S., 2000, "Optimization of Conditional Value-at-Risk," Journal of Risk, 2(3), pp.21-42.

[20] Beder, T.S, 1995, "VAR: Seductive but Dangerous," Financial Analysts Journal, 51(5), pp.12-24.

[21] Taylor, B.N., and Kuyatt, C.E., 1994, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST Technical Note 1297, U.S. Government Printing Office, Washington, DC.